LOCAL STRUCTURE OF A BINARY FINELY DISPERSED FLUIDIZED BED

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Results are given of calculations of the quantities characterizing the random pseudoturbulent motions of the phases in a homogeneous fluidized bed consisting of particles of two sorts, differing in size. The dependence of the coefficients of pseudoturbulent diffusion of the particles, the mean-square velocities of the pulsations, etc., on the partial concentrations of the particles, the ratio of their sizes, and other parameters is evaluated. For granular beds, fluidized by a gas or a drop-type liquid, intense chaotic fluctuations of both phases are characteristic; these determine to a considerable degree the observed macroscopic properties of the bed and affect its effectiveness as a working body in various types of heat exchangers and chemical reactors. Such random ("pseudoturbulent") motions are particularly considerable for beds of small particles under homogeneous fluidization conditions, where mixing due to the rise of cavities in the bed, filled only with the fluidizing medium, is practically absent. A similar situation is encountered in reactor and regenerating units for catalytic cracking [1, 2], in beds with a drop-type liquid phase, in rarefied two-phase systems under the conditions of strong fluidization or of the transport of bulk materials in a dilute phase, etc. The characteristics of pseudoturbulence in locally homogeneous flows of monodisperse two-phase systems have been investigated, for example, in [3-5]. However, real fluidized beds are generally polydisperse; the presence of particles of different sizes in the bed has a very considerable effect on the intensity of the pulsations, the effective diffusion coefficients of the phases of the bed, the effective viscosities, etc. [1, 6]. In addition, the chaotic mixing in polydisperse beds determines some of the technological characteristics, specifically, the rate of entrainment of small particles by the flow of the fluidizing medium and the settling of large particles, the degree of separation of the fractions of the disperse phase, which is very important in determination of the limits of the existence of the fluidized state, and in the modeling of numerous processes of the separation of particles with respect to size or density [1, 6].

§1. We consider a homogeneous fluidized bed of particles of radius a_j and density d_j (the subscript j denotes the sort of particles), assuming the Reynolds numbers, constructed with respect to a_j and the relative velocity of the liquid phase **u**, to be small. The latter permits assuming the interaction between the particles and the flow to be linear with respect to **u**, and using, for the force of the interaction f_j between a particle and the constrained flow in a bidisperse cloud of particles, the results of [7], in accordance with which we can write

$$f_j = 6\pi\mu a_j F_j, \quad F_1 = \varepsilon K(\alpha, \varkappa, \rho), \quad F_2 = \varepsilon K(\alpha^{-1}, \varkappa^{-1}, \rho), \quad \varepsilon = 1 - \rho,$$

$$K(\alpha, \varkappa, \rho) = 1 + \frac{3(5\varkappa + 3\alpha + 2\alpha^2)\rho}{2(2-3\rho)(1+\varkappa)} + \frac{27(\varkappa+\alpha)^2\rho^2}{2(1+\varkappa)^2(2-3\rho)^2} +$$

$$+ \frac{1}{2-3\rho}\left[1 + \frac{3(\varkappa+\alpha)\rho}{(2-3\rho)(1+\varkappa)}\right]\left[\frac{81(\varkappa+\alpha)^2\rho^2}{4(1+\varkappa)^2} + \frac{9(\varkappa+\alpha^2)(2-3\rho)\rho}{1+\varkappa}\right]^{1/2}. \tag{1.1}$$

Here the following parameters are introduced:

$$\varkappa = \rho_1/\rho_2, \quad \alpha = a_1/a_2, \quad \rho = \rho_1 + \rho_2, \tag{1.2}$$

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where ρ_j are the partial volumetric concentrations of particles of the j-th type; ϵ is the porosity of the bed; and μ is the viscosity of the liquid phase. The force of the interaction, referred to the particles in unit volume of the mixture, is obtained by multiplying f_j from (1.1) by the countable concentrations $n_j = \sigma_j^{-1}\rho_j$ (σ_j is the volume of particles of the j-th type).

The smallness of the Reynolds number makes it possible to assume that the system is "collisionless," in the sense that the interaction between the particles takes place mainly through random perturbations in the fields of the velocity and pressure of the liquid phase, while the role of indirect collisions in processes of momentum and energy transfer between particles is not great. It is then permissible to use the same model and the same stochastic equations for random pseudoturbulent quantities as in [3-5].

Neglect of direct collisions is also completely justified for particles of moderate size, right up to values of the Reynolds number on the order of 100. However, with such Reynolds numbers real fluidized beds are inhomogeneous. Therefore, the results obtained will characterize only pulsations in the so-called "dense" phase of the fluidized bed (outside of cavities, practically free of particles). With a further increase in the Reynolds number, the role of direct collisions in momentum and energy transfer is reinforced and, finally, becomes dominating, and the model in [3-5] ceases to be valid. An attempt to take account of collisions by introducing an integral term of the same form as that figuring in the Boltzmann equation for a dense gas into the kinetic equation was made in [8], where no account was taken of several important factors, characterizing the interaction between the particles and the liquid phase, and which are particularly significant precisely for large particles (for example, the Magnus force). It is obvious that a more perfect kinetic theory for a coarsely dispersed fluidized bed can be constructed on the basis of more modern concepts, and of the model, set forth in [9, 10].

In what follows, the concentrations ρ_1 and ρ_2 are assumed to be local characteristics of the bed, given a priori, and the question of their actual determination in different regions of the bed is not considered. To solve the problem posed, an investigation must be made of the macroscopic distribution of particles of both types over the height of the bed, under conditions of dynamic equilibrium, in the same way as an investigation was made in [5] of the distribution of the material in a monodisperse fluidized bed.

The characteristics of pseudoturbulence are calculated below in the first ("Euler") approximation, where ρ_j and the other dynamic parameters describing the macroscopic state of the system are regarded approximately as quantities which do not depend on the coordinates or the time, and which coincide with the local values in the region of the bed under investigation. A coordinate system is used in which the particles in the above region, on the average, are at rest, and the x_1 axis is directed along the mean velocity of the carrier flow $\mathbf{u}$. It is clear that the problem is symmetrical with respect to this axis.

Stochastic equations for the random pulsations of the velocity of the particles in the liquid phase and the pulsation of the pressures are obtained from the equations of motion in the same way as in [3-5]. Representing all the random quantities in the form of Fourier–Stieltjes integrals with the spectral measures ∂Z_{φ}, from the stochastic equations we can obtain a system of linear algebraic equations for the spectral measures, which have the form

$$im_j\omega d\mathbf{Z}_{\mathrm{w}}^{(j)} = 6\pi\mu a_j \left[F_j\left(d\mathbf{Z}_{\mathrm{v}} - d\mathbf{Z}_{\mathrm{w}}^{(j)}\right) + F_{j_1}\mathbf{u}dZ_{\rho}^{(1)} + F_{j_2}\mathbf{u}dZ_{\rho}^{(2)}\right]; \tag{1.3}$$

$$i\left(\omega + \mathbf{ku}\right)\left(dZ_{\rho}^{(1)} + dZ_{\rho}^{(2)}\right) - i\varepsilon\mathbf{k}d\mathbf{Z}_{\mathrm{v}} = 0;$$

$$id_0\varepsilon\left(\omega + \mathbf{ku}\right)d\mathbf{Z}_{\mathrm{v}} = -i\mathbf{k}dZ_p - 6\pi\mu a_1\rho_1/\sigma_1\left[F_1\left(d\mathbf{Z}_{\mathrm{v}} - d\mathbf{Z}_{\mathrm{w}}^{(1)}\right) + \right.$$

$$\left. + F_{11}\mathbf{u}dZ_{\rho}^{(1)} + F_{12}udZ_{\rho}^{(2)}\right] + 6\pi\mu a_2\rho_2/\sigma_2\left[F_2\left(d\mathbf{Z}_{\mathrm{v}} - d\mathbf{Z}_{\mathrm{w}}^{(2)}\right) + F_{21}\mathbf{u}dZ_{\rho}^{(1)} + F_{22}\mathbf{u}dZ_{\rho}^{(2)}\right],\ F_{\gamma\delta} = \frac{dF_{\gamma}}{d\rho_{\delta}},$$

where $m_j = d_j\sigma_j$ is the mass of a particle of the j-th sort; the coefficients F_j and ϵ are defined in (1.1); and ω and $\mathbf{k}$ are the frequency and the wave vector of the pulsations, respectively.

The system of equations (1.3) will be closed if the statistical characteristics of the spectral measures $dZ_{\rho}^{(j)}$ of the partial volumetric concentrations are known. For the spectral densities of the fluctuations of these concentrations we use expressions following from the theory in [11],

$$\Psi_{\rho,\rho}^{(j)}(\omega, \mathbf{k}) = \frac{\mathbf{k}D^{(j)}\mathbf{k}}{\pi}\frac{\Phi_j Y\left(\mathbf{k}^{(j)} - \mathbf{k}\right)}{\omega^2 + \left(\mathbf{k}D^{(j)}\mathbf{k} - T_j\omega^2\right)^2},$$

$$\Phi_j = \frac{3\rho_j\rho\left(1 - \rho/\rho_*\right)}{4\pi\mathbf{k}^{(j)3}},\ \mathbf{k}^{(j)} = (4.5\pi\rho)^{1/3}/a_j,\ T_j = \frac{\operatorname{tr} D^{(j)}}{\left\langle \mathbf{w}^{(j)\prime 2}\right\rangle}. \tag{1.4}$$

Here $D^{(j)}$ is the tensor of the effective pseudoturbulent diffusion of particles of the j-th type, diagonal

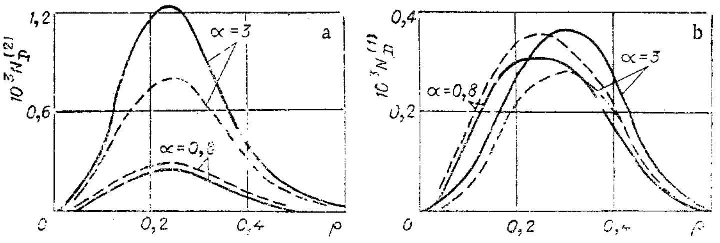

Fig. 1

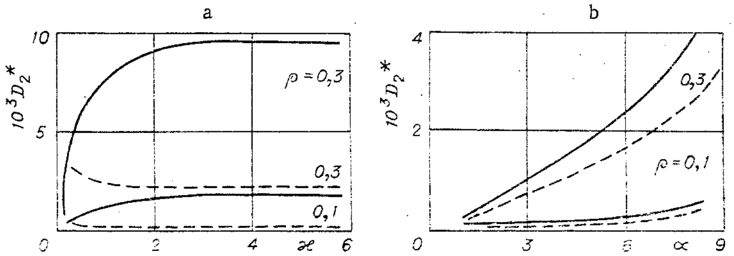

Fig. 2

in the selected system of coordinates; the coefficients of the transverse diffusion in both directions are equal.

To close (1.3), (1.4), analogously to [4, 5] we use a representation of the tensors $D^{(j)}$ and the quantities $\langle w^{(j)'2}\rangle$ in terms of the spectral density $\Psi^{(j)}_{w,w}$ of the pulsation velocities of the particles:

$$D_i^{(j)} = \pi/2 \int \Psi^{(j)}_{w_i w_i}(0, \mathbf{k})\, d\mathbf{k}, \langle w^{(j)'2}\rangle = \iint \mathrm{tr}\, \Psi^{(j)}_{w,w}(\omega, \mathbf{k})\, d\omega d\mathbf{k}. \qquad (1.5)$$

Equations (1.3), together with (1.4), (1.5), permit expressing all the spectral densities which are of interest in closed form.

§2. Expressing $dZ^{(j)}_w$ in terms of $dZ^{(j)}_\rho$ using (1.3), calculating from this the tensor spectral density $\Psi^{(j)}_{w,w}(\omega, \mathbf{k})$ taking account of relationships (1.4), assuming the frequency ω equal to zero, and integrating in the first relationship of (1.5), after calculations we obtain the following equations for the coefficients of longitudinal $D_1^{(j)}$ and transverse $D_2^{(j)} \equiv D_3^{(i)}$ pseudoturbulent diffusion of the particles:

$$\begin{gathered}
D_1^{(j)} D_2^{(j)} = 2\pi \mathbf{u}^2 \mathbf{k}^{(1)} \gamma_1^2 \Phi_1 \left[J_4^{(1)}/\varepsilon^2 + 2J_2^{(1)} L_{j1}/\varepsilon + L_{j1}^2 J_0^{(1)}\right] + \\
+ 2\pi \mathbf{u}^2 \mathbf{k}^{(2)} \gamma_2^2 \Phi_2 \left[J_4^{(2)}/\varepsilon^2 + 2J_2^{(2)} L_{j2}/\varepsilon + L_{j2}^2 J_0^{(2)}\right], \\
D_2^{(j)2} = \pi \mathbf{u}^2/\varepsilon^2 \left[\mathbf{k}^{(1)} \gamma_1^2 \Phi_1 \left(J_2^{(1)} - J_4^{(1)}\right) + \mathbf{k}^{(2)} \gamma_2^2 \Phi_2 \left(J_2^{(2)} - J_4^{(2)}\right)\right], \\
\gamma_j^2 = D_2^{(j)}/\left(D_1^{(j)} - D_2^{(j)}\right),\ J_n^{(j)} = \int_0^1 t^n/\left(t^2 + \gamma_j^2\right) dt,\ L_{j\delta} = \frac{\partial \ln F_j}{\partial \rho_\delta}.
\end{gathered} \qquad (2.1)$$

These equations are similar in form to the equations in [4, 5]. Eliminating the quantities $D_1^{(j)}$ and $D_2^{(j)}$ from (2.1), we obtain a system of two transcendental equations for determining the parameters γ_j:

$$\begin{gathered}
\gamma_1^2 J_4^{(1)}/\varepsilon^2 + 2\gamma_1^2 J_2^{(1)} L_{j1}/\varepsilon + \gamma_1^2 J_0^{(1)} L_{j1}^2 + \frac{1}{\alpha^2 \varkappa}\left[\gamma_2^2 J_4^{(2)}/\varepsilon^2 + 2\gamma_2^2 J_4^{(2)} L_{j1}/\varepsilon + \right. \\
\left. + \gamma_2^2 J_0^{(2)} L_{j2}^2\right] = \frac{\gamma_1^2\left(1+\gamma_j^2\right)}{2\varepsilon^2\gamma_j^2}\left(J_2^{(1)} - J_4^{(1)}\right) + \frac{\gamma_2^2\left(1+\gamma_j^2\right)}{2\varepsilon^2\gamma_j^2 \varkappa \alpha^2}\left(J_2^{(2)} - J_4^{(2)}\right);\ j = 1{,}2.
\end{gathered} \qquad (2.2)$$

It can be shown that, for any given set of values of the parameters $\varkappa$, α, and ρ having physical meaning, the system (2.2) has a single set of positive roots; $D_1^{(j)}$ and $D_2^{(j)}$ are expressed from (2.1) in the form

$$\begin{gathered}
D_i^{(j)} = \varepsilon a_2 \mathbf{u} D_i^{(j)*},\ D_2^{(j)*} = N_D^{(j)} D_1^{(j)*}, \\
D_2^{(j)*} = \left\{\frac{3}{4}\left(\frac{2}{9\pi}\right)^{2/3} \frac{\varkappa\alpha^2\rho^2(1-\rho/\rho_*)}{(1+\varkappa)\rho^{2/3}(1-\rho)^4}\left[\gamma_1^2\left(J_2^{(1)} - J_4^{(1)}\right) + \gamma_2^2\left(J_2^{(2)} - J_4^{(2)}\right)\right]\right\}^{1/2},\ N_D^{(j)} = \frac{\gamma_j^2}{1+\gamma_j^2}.
\end{gathered} \qquad (2.3)$$

From this it can be seen that the coefficient of transverse pseudoturbulent diffusion is identical for particles of both fractions, while the coefficients of longitudinal diffusion differ considerably. However, $D_2^* = D_2^{(j)*}$ from (2.3) depends on the roots γ_j of system (2.2) and, consequently, also on the fractional composition of the fluidized bed.

The solution of Eqs. (2.2) and the calculation of the quantities in (2.3) were carried out numerically on a BÉSM-4 digital computer. The dependences of the dimensionless coefficients from (2.3) on ρ have the same character as in a monodisperse bed [4, 5]. When ρ approaches zero or the concentration of the system, in a state of dense packing, ρ_*, these coefficients revert to zero; they all have maxima with $\rho = 0.2\ldots0.3$. The longitudinal diffusion is found to be considerably more intense than the transverse, which is also in agreement with the conclusions for a monodisperse bed in [4, 5].

However, the numerical values of the quantities $N_D^{(j)}$ and $D_i^{(j)*}$ depend to a considerable degree on the fractional composition of the bed, i.e., on the parameters α and $\varkappa$ from (1.2). Characteristic dependences of $N_D^{(1)}$ and $N_D^{(2)}$ on ρ with $\varkappa = 0.5$ (dashed curves) and $\varkappa = 2$ (solid curves) are shown in Fig. 1. The anisotropy of the pseudoturbulent diffusion is minimal with values of ρ corresponding to the maxima of the curves in Fig. 1, and depending very weakly on α and $\varkappa$. The absolute values of the coefficients $N_D^{(j)}$ for large (small) particles increases (decreases) considerably with an increase in their relative concentration, the more appreciably the greater the difference in the sizes of the particles.

Dependences of the dimensionless coefficient of transverse diffusion D_2^* on the relative volumetric concentration $\varkappa$ with $\alpha = 0.8$ (dashed curves) and $\alpha = 3$ (solid curves) are shown in Fig. 2a; Fig. 2b gives dependences of D_2^* on the parameter α with $\varkappa = 0.5$, 2 (dashed and solid curves, respectively). From Fig. 2 it can be seen that the dilution of a monodisperse system with larger particles considerably (by more than two times) intensifies the transverse turbulent mixing. On the contrary, the addition of relatively small particles to the system leads to a certain weakening of the diffusion in a transverse direction; under these circumstances, D_2^* is a monotonically increasing function of the ratio of the radius to the large particles to the radius of the small particles.

Analogous dependences were plotted also for the dimensionless coefficients of pseudoturbulent diffusion of particles of both fractions in a longitudinal direction. The character of these dependences is clear from the representation of $D_1^{(j)*}$ in the form of the product of D_2^* by the coefficient $N_D^{(j)-1}$ and the curves in Figs. 1 and 2. As an example, in Fig. 3a the value of $D_1^{(2)*}$ is shown as a function of $\varkappa$ with $\alpha = 0.8$, 3 (dashed and solid curves, respectively), and Fig. 3b gives the dependences of this same quantity on α with $\varkappa = 0.5$ (dashed curves) and $\varkappa = 2$ (solid curves). It can be seen that the polydisperse character of the bed has a serious effect on the rates of longitudinal diffusion; the overall character of the dependences in Fig. 3 is the same as that of the curves in Fig. 2. Specifically, the coefficient of longitudinal diffusion of large (small) particles is a monotonically increasing (decreasing) function of $\alpha (\alpha > 1)$; the rate of its change with α is greater the higher (lower) the relative concentration of the particles in the binary bed under consideration.

All these conclusions relate, of course, to systems characterized by an identical value of ρ, and differing only in their values of α and $\varkappa$.

§3. Equations (1.3)-(1.5) make it possible to find the spectral measures and the densities of all the random processes under investigation and to express the different correlation functions in the form of definite integrals with respect to the frequency of the pulsations ω and the wave space k, using standard methods. Calculations are made below of some of the most important characteristics of pseudoturbulence, giving a sufficiently true representation of the properties of the pseudoturbulence and of the effect of the fractional composition of the bed on them. For simplicity, we limit ourselves to an analysis of granular beds fluidized by a gas, assuming d_1 and d_2 are equal and using the ratio $\theta = d_0/d_1 \ll 1$ as a small parameter. Just such beds are typical for catalytic cracking units [2].

From (1.3), (1.4), using a standard method, we obtain formal representations for the mean squares of the components of the velocity of the particles in longitudinal and transverse directions, depending on the mean squares of the modulus of the velocity and the coefficients of the pseudoturbulent diffusion of the parti-

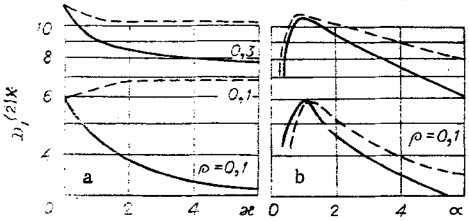

Fig. 3

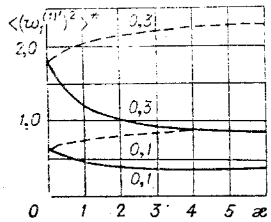

Fig. 4

cles as parameters. Omitting the details of the calculation, we write these representations for the particles of the first fraction:

$$\langle (w_1^{(1)'})^2\rangle^* = \frac{\langle (w_1^{(1)'})^2\rangle}{u^2} = \frac{27\theta\rho^2(1-\rho/\rho_*)}{(4.5\pi)^{2/3}\,\mathrm{Re}\,(1+\varkappa)\,\rho^{2/3}D_2^{(1)*}} \times$$

$$\times\left[\varkappa\gamma_1^2 J_4^{(1)}/\varepsilon^2 + \varkappa\gamma_1^2 L_{11}J_0^{(1)} + \frac{\gamma_2^2 J_4^{(2)}}{\alpha^2\varepsilon^2} + \frac{\gamma_2^2 J_0^2 L_{12}^2}{\alpha^2} + \frac{2\varkappa\gamma_1^2 L_{11}J_2^{(1)}}{\varepsilon} + \right.$$

$$\left. + \frac{2\gamma_2^2 L_{12}J_2^2}{\alpha^2\varepsilon}\right] + \frac{243\rho^2\theta^2(1-\rho/\rho_*)\,\varepsilon^2}{(4.5\pi)^{2/3}\,\mathrm{Re}^2(1+\varkappa)\,\alpha^2\rho^{2/3}}\left[\frac{\varkappa\left(1+2N_D^{(1)}\right)F_1^2 J_4^{(1)}}{\varepsilon^2\langle (w^{(1)'})^2\rangle^*\left(1-N_D^{(1)}\right)} + \right.$$

$$\left. + \frac{\varkappa F_{12}\left(1+2N_D^{(2)}\right)J_0^{(2)}}{\alpha^2\langle (w^{(2)'})^2\rangle^*\left(1-N_D^{(2)}\right)} + \frac{2\varkappa\left(1+2N_D^{(1)}\right)F_1F_{11}J_2^{(1)}}{\varepsilon\langle (w^{(1)'})^2\rangle^*\left(1-N_D^{(1)}\right)} + \frac{2\left(1+2N_D^{(2)}\right)F_1F_{12}J_2^{(2)}}{\varepsilon\alpha^2\langle (w^{(2)'})^2\rangle^*\left(1-N_D^{(2)}\right)}\right]; \qquad (3.1)$$

$$\langle (w_2^{(1)'})^2\rangle^* = \frac{\langle (w_2^{(1)'})^2\rangle}{u^2} = \frac{\langle (w_3^{(1)'})^2\rangle}{u^2} = \frac{13.5\rho^2\theta(1-\rho/\rho_*)\,F_1}{(4.5\pi)^{2/3}\,\mathrm{Re}\,(1+\varkappa)\,\rho^{2/3}\varepsilon} \times$$

$$\times\left\{\varkappa\left[J_2^{(1)} - J_4^{(1)}\right]\left[\frac{\gamma_1^2}{\varepsilon D_2^{(1)*}} + \frac{9\theta F_1\left(1+2N_D^{(1)}\right)\varepsilon u^2}{\mathrm{Re}\alpha^2\langle (w^{(1)'})^2\rangle^*\left(1-N_D^{(1)}\right)}\right] + \frac{J_2^{(2)}-J_4^{(2)}}{\alpha^2}\left[\frac{\gamma_2^2}{\varepsilon D_2^{(1)*}} + \frac{9\theta F_1\left(1+2N_D^{(2)}\right)\varepsilon u^2}{\mathrm{Re}\alpha^2\langle (w^{(2)'})^2\rangle^*\left(1-N_D^{(2)}\right)}\right]\right\},$$

$$F_{\gamma\delta} = \frac{dF_\gamma}{d\rho_\delta},\ L_{j\delta} = \frac{\partial \ln F_j}{\partial\rho_\delta},$$

where the Reynolds number Re is introduced, defined in the following manner: $\mathrm{Re} = 2ud_0a_2(1-\rho)/\mu = 2ud_0a_1\cdot(1-\rho)/\alpha\mu$.

Summing expression (3.1) in accordance with the second relationship of (1.5), we obtain the equation

$$\langle (w^{(1)'})^2\rangle = A_1 + \frac{B_1}{\langle (w^{(1)'})^2\rangle} + \frac{C_1}{\langle (w^{(2)'})^2\rangle}, \qquad (3.2)$$

$$A_1 = \frac{6(9\pi)^{1/3}\theta\rho^{4/3}(1-\rho/\rho_*)\,F_1}{\pi\,\mathrm{Re}\,(1+\varkappa)\,2^{1/3}D_2^{(1)*}}\left[\frac{\varkappa\gamma_1^2 J_2^{(1)}}{\varepsilon^2} + 2\gamma_1^2\varkappa L_{11}J_2^{(1)}/\varepsilon + \frac{2\gamma_2^2 L_{12}J_2^{(2)}}{\alpha^2\varepsilon} + \frac{\gamma_2^2 J_2^{(2)}}{\alpha^2\varepsilon} + \varkappa\gamma_1^2 L_{11}^2 J_0^{(1)} + \gamma_2^2 L_{12}^2 J_0^{(2)}/\alpha^2\right]u^2,$$

$$B_1 = \frac{54(4.5\pi)^{1/3}\varkappa\theta^2\rho^{4/3}(1-\rho/\rho_*)\,\varepsilon^2\left(1+2N_D^{(1)}\right)}{\pi\mathrm{Re}^2\alpha^2(1+\varkappa)\left(1-N_D^{(1)}\right)}\left[F_1^2 J_2^{(1)}/\varepsilon^2 + 2F_1F_{11}J_2^{(1)}/\varepsilon + F_{11}^2 J_0^{(1)}\right]u^4,$$

$$C_1 = \frac{54(4.5\pi)^{1/3}\theta^2\rho^{4/3}(1-\rho/\rho_*)\,\varepsilon^2\left(1+2N_D^{(2)}\right)}{\pi\,\mathrm{Re}^2(1+\varkappa)\,\alpha^4\left(1-N_D^{(2)}\right)}\left[F_1^2 J_2^{(2)}/\varepsilon^2 + 2F_1F_{12}J_2^{(2)}/\varepsilon + F_{12}^2 J_0^{(2)}\right]u^4.$$

The expressions for the quantities $\langle (w_i^{(2)'})^2\rangle$, characterizing the particles of the second fraction, have the same form as in (3.1). They can be written by analogy to (3.1), taking into consideration that $\varkappa$, α, and Re must be replaced by $\varkappa^{-1}$, α^{-1}, and αRe, respectively. Summation of these quantities leads to an equation for $\langle (w^{(2)'})^2\rangle$, analogous to (3.2). Solution of these two equations makes it possible to find the mean squares of the modulus of the velocity for particles of both fractions as functions of the physical and operating parameters of the bed and, finally, to close the relationships (1.3)-(1.5).

Actual calculations were made with different values of the parameters Re and $\theta = d_0/d_1$, corresponding to fluidized beds encountered in reactor and regenerator catalytic cracking units. As an example, Figs. 4 and 5 give data on the longitudinal pulsation of the velocity, obtained with Re = 0.5 and $\theta = d_0/d_1 = 0.00125$. Figure 4 illustrates the dependences of the dimensionless quantity $\langle (w^{(1)'})^2\rangle^*$ on $\varkappa$ with $\rho = 0.1$ or 0.3 (numbers on curves) and $\alpha = 0.8$ (dashed curves) or $\alpha = 3$ (solid curves). Figure 5 gives dependences of the same quantity on α with $\varkappa = 0.5$, 2 (dashed and solid curves, respectively). As can be seen from Fig. 4, the pulsations of small (large) particles are reinforced (weakened) with an increase in their relative concentration, the more appreciably the higher the total volumetric concentration of the disperse phase.

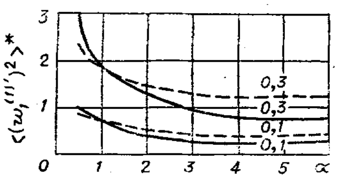

Fig. 5

Analogously, we can obtain expressions also for the other characteristics of pseudoturbulent motion; in the general case, these have a very cumbersome form. The representations for the mean squares of the longitudinal and transverse velocities of the gas have the simplest form

$$\langle v_1'^2\rangle = \frac{\rho^2(1-\rho/\rho_*)}{15(1+\varkappa)\varepsilon^2}\left[\frac{\varkappa\left(3+2N_D^{(1)}\right)}{1+2N_D^{(1)}}\langle (w^{(1)\prime})^2\rangle + \left(3+2N_D^{(2)}\right)\langle (w^{(2)\prime})^2\rangle / \left(1+2N_D^{(2)}\right) + 3(1+\varkappa)u^2\right];$$

$$\langle v_2'^2\rangle = \langle v_3'^2\rangle = \frac{\rho^2(1-\rho/\rho_*)}{15(1+\varkappa)\varepsilon^2}\left[\frac{\varkappa\left(1+4N_D^{(1)}\right)}{1+2N_D^{(1)}}\langle (w^{(1)\prime})^2\rangle + \frac{1+4N_D^{(2)}}{1+2N_D^{(2)}}\langle (w^{(2)\prime})^2\rangle + (1+\varkappa)u^2\right]. \quad (3.3)$$

and for the additional pulsation flow of the gas

$$\langle \rho_1' v_1'\rangle = \frac{\varkappa\rho^2(1-\rho/\rho_*)}{3(1+\varkappa)\varepsilon}u, \quad \langle \rho_2' v_1'\rangle = \frac{\rho^2(1-\rho/\rho_*)u}{3(1+\varkappa)\varepsilon}. \quad (3.4)$$

The character of the dependence of these quantities on the parameters of the bed can be brought out on the basis of relationships (3.3), (3.4), and the curves in Fig. 1.

As follows from the above analysis, the relationships between the particle sizes of both fractions and their contents in the disperse phase has a very considerable effect on the intensity and the properties of the pseudoturbulent pulsations; the character of this effect depends on the values of the other parameters of the bed. A change in the level of the development of the pseudoturbulence leads to corresponding changes in the character and the intensity of the mixing of the phases in a fluidized bed and, in the final analysis, has an effect on the effectiveness of the use of the bed in the organization of heat- and mass-transfer processes in industrial chemical reactors and other apparatuses; this must be taken into consideration in calculation and design of this equipment. In this respect, the conclusion of the existence of a weakening of the pulsations and a lowering of the effectiveness of the mixing with the addition of a small (by volume) amount of small particles to the original monodisperse bed is of great importance. The conclusion has been confirmed experimentally by known experiments on the dependence of the viscosity of fluidized systems on the composition of their disperse phase (see, for example, [1, 7, 12]), in accordance with which comparatively small additions of fines considerably lower the viscosity. The other properties of the model, presented above, related to the dependences of the mean-square velocities of the pulsations and the coefficients of the pseudoturbulent diffusion of the particles on $\varkappa$ and α, are, on the whole, confirmed by certain observations with beds used in catalytic cracking processes. However, the present state of experimental investigations of binary and other polydisperse fluidized beds does not permit any kind of detailed comparison between theoretical conclusions and experimental data.

The latter is rendered difficult to a considerable degree also as a result of the complexity and cumbersomeness of the theory itself, which is extremely unsuitable for use for practical purposes. Therefore, there is a need for the creation of a simplified theoretical model and engineering variants of the analysis of binary beds. In view of this, calculated data of the type given above are found useful both for clarifying the most significant factors, determining the behavior of a polydisperse bed under these or other conditions, and in verifying the conclusions following from such simplified schemes.

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